**CORE 203 – GREAT IDEAS OF MODERN MATHEMATICS – DR. NARDO**

**FALL 2015 – TEST 3 – VOTING THEORY – TTH 9:45 SECTION**

Printed Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Signature: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Your signature reaffirms your acceptance of the Oglethorpe Honor Code; it certifies that you have acted honorably on this test.

* **It is expected that you will not only give answers but also explain fully why those answers are correct! This has been the operating procedure in our class all semester.**
* If an explanation is not needed, then this will be explicitly noted in a problem.
* You must use correct mathematical symbols and correct mathematical terms/definitions.
* Problem #1 is worth 10 points, and each of the other problems is worth 18 points.

**1.** Define each of the terms below.

**A.** Majority Threshold **(I am NOT looking for a formula here!)**

**B.** Majority Candidate

**C.** Condorcet Candidate

**D.** Condorcet Property

**E.** Monotonicity Property

**2.** One hundred students were asked to rank their preferences for bottled water brands with the results below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Aquafina | 1 | 1 | 3 | 3 |
| Dasani | 2 | 3 | 1 | 2 |
| Evian | 3 | 2 | 2 | 1 |
| Number of Votes | 22 | 17 | 31 | 30 |

**A.** Calculate the majority threshold. **Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**B.** Give the vote totals below, showing any sums where needed and not just final “answers.”

|  |  |
| --- | --- |
| # of first-place votes for Aquafina = |  |
| # of first-place votes for Dasani = |  |
| # of first-place votes for Evian = |  |

**C.** Who is the winner under the plurality voting system? (If none, then write none.)

**Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**D.** Who is the majority candidate? (If none, then write none.)

**Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**E.** Is this problem a valid counter-example which shows that the plurality voting system does **NOT** possess the majority property? (Yes or No)

**Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**3.** In ramping up for the new “Star Wars” movie “The Force Awakens,” seventeen fans ranked their preferences for four of the franchise’s movies with the results below.

**S > E > J > R: 6 E > S > J > R: 5 J > R > E > S: 4 R > J > S > E: 2**

 NOTE: S = “Star Wars” E = “Empire Strikes Back”

 J = “Return of the Jedi” R = “Revenge of the Sith”

**A.** Calculate the majority threshold. **Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**B.** Give the vote totals below. (**No explanations are needed for Part B.)**

|  |  |
| --- | --- |
| # of first-place votes for S = | # of first-place votes for E = |
| # of first-place votes for J = | # of first-place votes for R = |

**C.** Which film is the winner under the plurality with elimination voting system?

(If none, then write none.)

**Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**4.** A student club must decide what to do with left-over funds from the fall semester. There are three choices, and the club members rank their preferences as below.

|  |  |  |  |
| --- | --- | --- | --- |
| Proposal A: Refund to Students | 1 | 3 | 3 |
| Proposal B: Throw a Party for Club Members | 2 | 1 | 2 |
| Proposal C: Save the Fund for the Spring Semester | 3 | 2 | 1 |
| Number of Votes | 55 | 50 | 3 |

**A.** Who is the winner under the Borda Count voting system?

(If none, then write none.) **Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**B.** Who is the Condorcet candidate? (If none, then write none.)

**Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**C.** Is this problem a valid counter-example which shows that the Borda Count voting system does **NOT** possess the Condorcet property? (Yes or No)

**Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**5.** You ask customers at the OU Starbucks to rank three beverage choices in popularity.
 E = Hot Espresso Drinks F = Frappuccino Drinks T = Tea Drinks

 Your sample of customers gives the preference data below.

 **E > F > T: 25 T > F > E: 20 F > T > E: 15**

**A.** Calculate the majority threshold. **Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**B.** How is the majority threshold used in the Pairwise Comparisons voting system?

**C.** Who is the winning drink under the Pairwise Comparisons voting system?

**Winner: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation: Fill in the table below and then add any needed explanation afterwards.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Pairwise Race #1 | Pairwise Race #2 | Pairwise Race #3 |
| First Candidate: |   |  |  |
| Second Candidate: |  |  |  |
| Three Modified Preference Inequalities with Vote Totals: |  |  |  |
| # of Votes for First Candidate = |  |  |  |
| # of Votes for Second Candidate = |  |  |  |
| Pairwise Winner = |  |  |  |
| How were Condorcet Points awarded to all candidates in this pairwise race? |  |  |  |

**D.** You are disappointed that your favorite drink was not the winner and explore this voting data more closely. Show that a different voting system can give a different winner here.

**New Winner: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**New Voting System: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Explanation:

**6.** Consider the following weighted voting system: {19: 19, 17, 2}.

**A.** List all coalitions. **(No explanations are needed for Part A.)**

**B.** Make a table (like in class) that shows all winning coalitions and the critical voters for each winning coalition. **(No explanations are needed for Part B.)**

**C.** From Part B, pick a winning coalition and a critical voter from that coalition. Explain why the coalition is a winning one and why the voter is critical, by class definitions.

 Critical Voter: \_\_\_\_\_\_\_\_\_\_\_\_\_

 Winning Coalition: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Explanation:

**(Continued)**

**6. (Continued)**

**D.** Give the definition of the Banzhaf power index. You should use generic voter *v*.

BPI(*v*) =

**E.** Compute the Banzhaf power index for each voter. Give a reduced fraction “answer.”

**F.** At first glance, it might appear that voter A is a dictator in this weighted voting system since he/she has a weight equal to the quota. Explain briefly why A is not a dictator.

**BONUS – 8 POINTS**

As seen in class, only one of our four voting systems possesses the Condorcet Property. Identify that voting system, and give a proof of this claim.