ENGAGING MATHEMATICS

CREATING A COMMUNITY OF PRACTICE THAT ENGAGES STUDENTS IN MATHEMATICS THROUGH MEANINGFUL CIVIC APPLICATIONS

An initiative of the National Center for Science and Civic Engagement, Engaging Mathematics applies the well-established SENCER method to college level mathematics courses, with the goal of using civic issues to make math more relevant to students.

Engaging Mathematics will: (1) develop and deliver enhanced and new mathematics courses and course modules that engage students through meaningful civic applications, (2) draw upon state-of-the-art curriculum in mathematics, already developed through federal and other support programs, to complement and broaden the impact of the SENCER approach to course design, (3) create a wider community of mathematics scholars within SENCER capable of implementing and sustaining curricular reforms, (4) broaden project impacts beyond SENCER by offering national dissemination through workshops, online webinars, publications, presentations at local, regional, and national venues, and catalytic site visits, and (5) develop assessment tools to monitor students' perceptions of the usefulness of mathematics, interest and confidence in doing mathematics, growth in knowledge content, and ability to apply mathematics to better understand complex civic issues.

Updates and resources developed throughout the initiative will be available online at www.engagingmathematics.net. Follow the initiative on Twitter: @MathEngaging.

Industrial Stormwater and Statistics

Dr. Cindy Kaus
Associate Professor
and Chair, Mathematics
cindy.kaus@metrostate.edu
Engaging Mathematics
Co-Principal Investigator

Partnering with the Minnesota Pollution Control Agency to Teach Introductory Statistics

<u>Courses</u>

Statistics I: STAT 201 is an introductory statistics course that meets the general education requirement in mathematics at Metropolitan State University.
» Content covers descriptive and inferential statistics (hypothesis testing, two-sample inference, least squares regression).
» One section of STAT 201 with an emphasis on environmental issues will be taught for the first time in the spring semester of 2015.
» The course will emphasize environmental issues with group work, homework, and examples generated from environmental data.
» Students will complete semester-long group projects analyzing data for the Minnesota Pollution Control Agency.

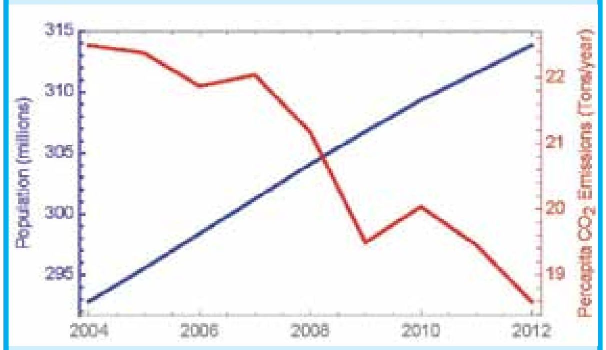
Who Takes this Course?
» Prerequisites for the course are *Intermediate Algebra* or appropriate placement at college level math on the university's math assessment.
» Students' majors include business, criminal justice, mathematics, natural sciences, nursing, psychology, social science, and social work.

Environmental Statistics: STAT 351 is an upper division statistics course that can be used as an elective in the statistics minor at Metropolitan State University.
» Content covers T-tests, F-tests, ANOVA, MANOVA, correlation and regression (simple and multiple), principle component analysis, and cluster analysis.
» STAT 351 will be taught for the first time in the spring semester of 2016.
» The course will also incorporate semester-long group projects analyzing data for the Minnesota Pollution Control Agency.

Who Takes This Course?
Students in STAT 351 come from a wide range of academic backgrounds and majors.
» Prerequisites for the course are STAT 201 (*Statistics I*) and STAT 251 (*Statistics Programming I*).
» Metropolitan State University has a minor in statistics that attracts students from many disciplines including mathematics, computer science, natural sciences, psychology, and nursing.

<u>How are These Courses Engaging?</u>
Students will be partnering with the Minnesota Pollution Control Agency (MPCA) Industrial Stormwater Division to analyze data obtained from stormwater collections by 29 different industrial sectors and 45 subsectors. The students in both courses will have the opportunity to enrich their academic experience through this collaborative effort with the MPCA.

Sustainability Models for Algebra Courses



U.S. population and per capita carbon dioxide emissions from energy consumption for the period 2004-2012.

Dr. Rikki Wagstrom
Associate Professor,
Mathematics
rikki.wagstrom@
metrostate.edu

Metropolitan State University offers the course *Mathematics of Sustainability* as an intermediate algebra alternative to serve students with above-average mathematical skills who have been away from mathematics for some time. The course curriculum includes a series of modules spanning a range of sustainability-related topics that reinforce mathematical content throughout the course. Though developed for use in a non-standard mathematics course, the modules are self-contained, portable units appropriate for use in college algebra, pre-calculus, and liberal arts mathematics courses as well. The following modules are new additions to the curriculum:

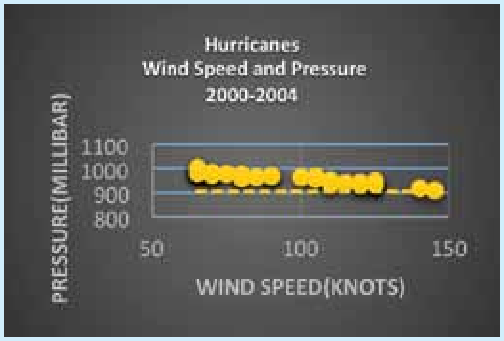
Milkweed: What about it? and The Monarch and the Milkweed: This two-part module explores disappearing milkweed populations in the U.S. and the potential impact on monarch butterfly populations. Mathematical content emphasized: quantitative literacy topics (scientific notation, units, density, percent change, reading graphs), and exponential and logarithmic functions.

How Much Energy Does a Wind Turbine Produce? and Wind Energy: What does it Cost?: The first part of this two-part module introduces students to the basic concepts of energy and power and their units of measurement, and goes through the derivation of a mathematical model for the power generated by a wind turbine over small periods of time, which students use to estimate energy generation. The second part of this module explores a range of scenarios impacting the profitability of wind energy development. Mathematical content emphasized: quantitative literacy topics (units and unit conversion), and derivation and evaluation of multivariable functions.

Liquid Fuel Comparisons: This module, co-authored with Metropolitan State University math education student Jodin Morey, explores greenhouse gas emissions associated with different automotive fuels; in particular conventional gasoline, ethanol blends, diesel, and biodiesel. Mathematical content emphasized: quantitative literacy topics (unit conversion, percent change, rate of change), and derivation and evaluation of multivariable functions.

Exploring Carbon Dioxide Emissions in the United States: This module explores the interplay between population growth in the U.S. and declining per capita carbon footprints. Modeling both of these trends using linear functions creates a quadratic model for total carbon emissions in the U.S., which appears to fit recent trends well. This model may be used for short-term extrapolation, but becomes problematic farther out in time as per capita emissions reach impossibly low levels and linear population growth becomes questionable. Mathematical content emphasized: quantitative literacy topics (reading graphs, units, and unit conversion), linear and quadratic functions, and extrapolation issues.

Elementary Statistics: Society and Environment



Dr. Mangala Kothari
Associate Professor,
Mathematics
mkothari@lagcc.cuny.edu
Engaging Mathematics
Co-Principal Investigator

Contributors:
Engaging Mathematics Team at LaGuardia

Dr. Alioune Khoule
Faculty Fellow,
Assistant Professor, Mathematics
Dr. Marina Nechayeva
Associate Professor, Mathematics
Dr. Nana Bonsu
Assistant Professor, Mathematics
Dr. Steven Cosares
Assistant Professor, Mathematics
Dr. Milena Cuellar
Assistant Professor, Mathematics
Brian Johnston
Quantitative Reasoning Fellow, CUNY

The modules are tailored for students required to complete a standard elementary statistics course as part of their major or as part of an elective math course sequence. Modules are designed to be portable units that may be implemented within the standard curriculum as in-class activities, in-class group work, or group research projects. Each includes a variety of topics related to the overarching theme of society and environment that address the underlying course concepts.

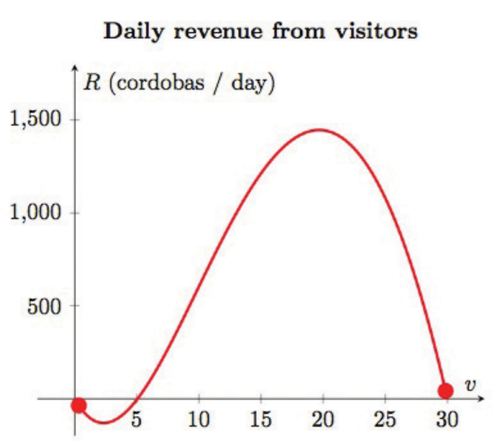
Module 1: Data Collection, Sampling Methods, and Descriptive Statistics: This module introduces basic statistical concepts such as data collection, sampling techniques, graphical representation, and organization and summation of data. In the demographics activity, students are presented with the fundamental question: How can we describe a group of people? Students design a sample framework to collect data for variables such as country of origin, age, gender, and NYC borough of residence. Students then generate a report based on their observational study. In this experience-based assignment, students learn through practice about sampling techniques, bias, and population data. The module concludes with a comparative demographic study of neighborhoods, the college, and the city.

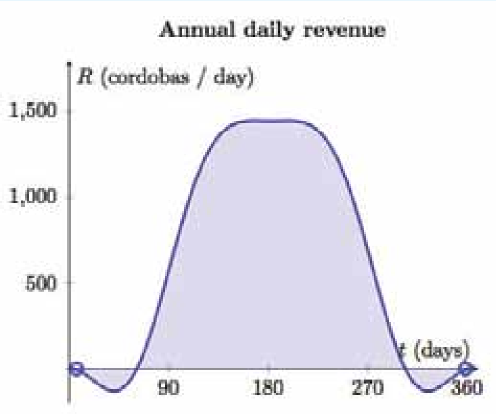
Module 2: Describing Relationships between Two Variables: Correlation and Regression: Students learn to identify linear correlations between two data sets using scatterplots and the linear correlation coefficient. In addition, they learn to identify causation and its limits when analyzing real data. For example, the instructor provides data on maximum sustained wind speeds and the pressure within a hurricane from 2000 to 2004. Students are asked to identify if there exists a possible linear relationship between wind speeds and pressure within a hurricane, develop a model that describes this relationship, and use statistical analysis to estimate wind speed in a hurricane with a given pressure. Activities in module 1 can also be extended to this module.

Module 3: Probability and Probability Distributions: Fundamental concepts of probability are presented to students through two-way tables compiled from the 2010 Census and NYC agencies reporting neighborhood composition changes over time, and how air and water affects NYC residents' health. In another activity, students are presented with a two-way table compiled using data from 1896-2012 presidential elections regarding the number of times the winning candidate was taller or shorter, and heavier or lighter, than the opponent. Students compute the probabilities to determine if height and weight play a role in political success.

Module 4: Inferential Statistics: Students revisit data analysis, estimate confidence intervals, and learn the fundamentals of hypothesis testing. In the preparatory stage, students consider their family, friends, and acquaintances and decide if there is any indication that their generation is different in height from their parents'. They state their informal claim, and whatever evidence they have for making it. In the activity's successive stages, students gather their own data or use sample data provided to conduct descriptive statistics and hypothesis testing.

Sustainability Analysis of a Rural Nicaraguan Coffee Cooperative





Sample results are shown in the above plots using data provided from the cooperative. The minimum number of visitors to make the cooperative profitable (R > 0) can be estimated. Additional extensions could include the effect of different pricing schemes and discounts on the cooperative's revenue.

Dr. John Zobitz
Associate Professor,
Mathematics
zobitz@augsburg.edu

A coffee cooperative in the Peñas Blancas region of northern Nicaragua is seeking to evaluate additional revenue streams to offset any losses due to shortcomings in the annual coffee harvest. This module examines multi-faceted aspects of sustainability through analysis of resource allocation, revenue streams, and long-term sustainability for ecotourism development.

A basic model for revenue production R is the following:

$$R = p \cdot v - k \cdot v - C$$

R = Revenue / day (cordobas / day). Cordobas are the Nicaraguan currency.
p = Visitor charge (cordobas) to stay at the cooperative.
v = The number of daily visitors (visitors / day).
k = Daily cost (cordobas / visitor) to accommodate guests.
C = The fixed costs in the upkeep of the cooperative (cordobas / day) to accommodate visitors.

This module can be implemented in a calculus course with a variety of topics:

• **Modeling:** What is a reasonable model for the number of daily visitors to the cooperative over the course of the year? What is the minimum price p that can be set for the cooperative to have a positive revenue?

• **Parameter estimation:** Given a table of items used by visitors and their cost, estimate a value of k and C to set the minimum price p for the cooperative to be profitable.

• **Related rates:** Over the course of a year, how does the cooperative's daily revenue change according to the number of visitors?

• **Optimization:** Use search engine analytics to determine times of the year the cooperative should aggressively market itself to attract more visitors.

• **Differential equations:** What is a basic model for customer acquisition and how is that translated into the cooperative's revenue?

• **Integration:** What is the annual revenue for the cooperative?

Groundwater Pollution Module

Darcy's law, relates the *flux* q, which is the discharge rate per unit cross-sectional area, with the gradient ∇h of the hydraulic head

$$q = -K\nabla h, \quad (1)$$

where K is the *hydraulic conductivity*. The fluid net velocity called *interstitial velocity* or, simply, velocity of the ground water, is given by the following formula

$$v = \frac{-K\nabla h}{\eta}, \quad (2)$$

where η denotes the *porosity* of the medium.

Figure 1: Darcy's law

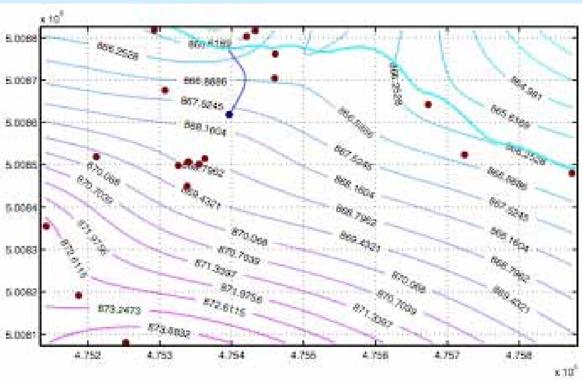


Figure 2: Water table contour map showing a contaminated groundwater flow line from a monitoring well to a nearby creek. This is a MATLAB graphical output from a project based on a municipal landfill located in the city of Andover, Anoka County, Minnesota.

Dr. Victor Padron

Instructor, Mathematics

victor.padron@
normandale.edu

Who Uses the Module

The module has been tailored for students in the standard calculus sequence, including pre-calculus or a mathematical modeling course, and was designed as a portable unit that could be implemented within the standard curriculum as individual or group research projects.

Aim of Module

The aim of the module is to provide a self-contained introduction to the subject and to engage students in the process of using mathematical models to investigate real events of groundwater pollution.

Framework of Modeling Groundwater

A typical groundwater-modeling framework involves considering a flow that is essentially horizontal, moving in either of the two directions of an imaginary plane that is "parallel" to the Earth's surface, and assuming that the height h of the water level at any point in the aquifer, known as the *hydraulic head*, is a function of the variables x and y representing the two dimensions of the flow field. The values h on an aquifer are fundamental to determining the direction and velocity of the flow, according to Darcy's law (see Figure 1).

Module Implementation

The module can be implemented at different levels of instruction. Students in a pre-calculus course use a discrete version of equations (1)-(2) of Darcy's law to track groundwater pollutants and estimate their traveling time between different locations. In a second semester of calculus, students could be presented with realistic examples from which contaminated groundwater flow lines could be obtained analytically by solving a separable differential equation. More advanced students could use numerical methods to handle real events of groundwater pollution (see Figure 2).

Teaching General Education Mathematics with Watershed

Dr. Anthony Dunlop
Instructor, Mathematics
anthony.dunlop@
normandale.edu

Who Takes the Course
This is a general education math course intended for non-STEM majors. Most of the students are humanities or professional studies (e.g., law enforcement, social work, etc.) majors.

Prerequisites/Required Background
Students who place into college algebra or above may take the course. An alternative prerequisite is a developmental math course designed at Normandale for students not intending to take college algebra and above. Assumed background knowledge includes graphing and solving linear equations; introduction to polynomials, including simple factoring; graphing quadratic, exponential, and logarithmic functions; and solving quadratic, exponential, and logarithmic equations. In practice, the level of preparation among the students varies widely. Prerequisite concepts are re-introduced and reviewed as needed.

The Approach
Mathematical topics covered are introduced on an "as needed" basis as we look at watershed data.

Old "Math for Liberal Arts" Model
The old model involved introducing a mathematical topic and then following up with "applications" or "examples" that were often contrived, artificial, or downright silly.

New "Quantitative Literacy" Model
The new model involves raising a question about water quality or management, looking up available data, and introducing mathematical or statistical techniques as needed to address a question.

Source of Data
Data from official publications of the Nine Mile Creek Watershed District (NMCWD), Hennepin County, MN, inform the course. Some assigned readings are taken from these publications as well.

Structure of the Course
No textbook is used. Some reading is assigned from NMCWD official publications (so students understand the ecological background and see the kind of mathematics present in such literature), from web sites (for basic statistics), and from materials written by the instructor (mostly for in-class activities).

The modules developed for the course have mathematical topics that could fit into a remedial algebra, college algebra, or introductory statistics course.

Great Ideas of Modern Mathematics

Targeted Course

As part of Oglethorpe University's nine-course, integrated, interdisciplinary core curriculum, the course *Great Ideas of Modern Mathematics* (GIMM) strives to teach students to think like mathematicians. Though it shares a spirit with liberal arts mathematics (LIB) courses taught at most universities in the country, it has some striking differences.

The mathematics explored in the course must be modern, defined as being created since the time of Sir Isaac Newton. This is meant to reinforce that new mathematics is being created every day and that our subject is not the static body of knowledge that some students mistakenly assume it to be. This is also meant to give students a break from the older, classical branches of mathematics mainly studied in our K-12 curriculum. Many times, students unsuccessful with those high school topics find a renaissance with the different fields of mathematics presented in the course.

Also, the topics chosen for the course are limited to three units, each called a "great idea," which are covered at a deep level. Thus, this course is not the typical "survey" course covering more topics but in less depth. Two topics are standard across all sections (probability and formal logic), and the third topic is the instructor's choice. Number theory, sets and infinity, group theory, and knot theory are some topics that have been taught in the past.

Project

This project focuses on creating curricular modules that can be used as third "great ideas" in GIMM. Of course, the modules, or even individual assignments, can also be used in more traditional courses. The two proposed topics are the mathematics of voting and the mathematics of poverty and income disparity. The first topic, further along in development, is described fully on the following page, and the second, using the more familiar tools of Riemann sums and correlation/regression, is slated for development in spring 2015.

Photographs at left are from the mentor walk that occurred at Oglethorpe, bringing K-12 students to campus to be paired with college students. It was attended by former President Carter and his wife, along with local Atlanta community and business leaders.

Dr. John C. Nardo
Professor, Mathematics
jnardo@oglethorpe.edu

Dr. Lynn Gieger
Associate Professor,
Mathematics
lgieger@oglethorpe.edu

The Mathematics of Voting

This module opens with four writing assignments. The initial assignment prompts students to list political issues important to them (at the national, state, or local level). Then, in an exploration of the applicability of their coursework to the real world, they must decide whether each issue can be explored using science or mathematics. In the third assignment, students grapple with the difficulties that can arise in elections with more than two candidates through the use of a newspaper article and professional journal article. Lastly, students brainstorm about the properties needed for an election to be "fair." In the follow-up class meeting, the students discuss and debate these writing assignments as a group.

After this rather non-traditional opening for a mathematics topic, the unit moves into a more traditional approach of covering new material in sections (as free electronic documents so that no textbook is needed) with homework. Basic voting definitions and the fairness criteria developed by Kenneth Arrow are introduced. Then students explore the various voting methods: plurality, plurality with elimination, Borda count, and pairwise comparisons. This part of the unit culminates with Arrow's Impossibility Theorem: no voting method with three or more candidates can satisfy all the given fairness criteria. Students are then able to debate the aftermath of such a startling result. Knowing that no method possesses all these desirable characteristics, they must argue the method they would use. This amount of material would be appropriate for one chapter in a more traditional LIB course.

For a GIMM course, there is more time to spend on this material, so students delve deeper within the theme to explore the problem of apportionment (how to divide a representative body based on the underlying units' populations). The motivating example of this will be the U.S. House of Representatives. Students will learn both historical apportionment methods and modern ones. After exploring apportionment paradoxes, they will again debate fairness, culminating in another devastating impossibility theorem. This "great idea" allows students to explore mathematical definitions, explanations, proofs, and counter-examples, and to apply those concepts to their civic world. Thus, we hope they will see there is more to voting than simply counting ballots.

College Algebra: Modeling the City

Dr. Bárbara González
Associate Professor
and Chair, Mathematics
and Actuarial Science
bgonzález@roosevelt.edu
(right)

Ms. Cathy Evins
Lecturer, Mathematics
cevins@roosevelt.edu
(left)

We will develop a new college algebra course that couples math concepts with important social justice challenges in the city of Chicago, which may include themes such as transportation, crime, water, food access, infrastructure, and demographics. The course will be "flipped," meaning that students will learn basic algebra skills outside of class, and then spend class time applying those skills to come up with solutions to problems derived from the themes. Eventually, the course may be split into two versions: one for STEM majors and the other for business majors. Ultimately, educators based in other cities may use the resources created for Chicago as a model to create similar classes at their own institutions. In order to understand how each of these components affects student learning, we will implement the changes in stages.

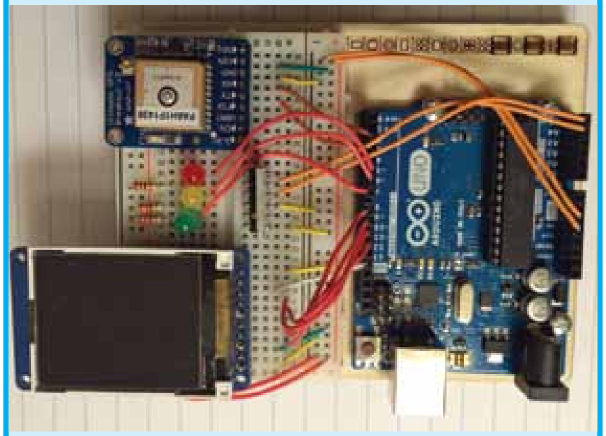
Stage 1: In fall 2014, we will offer two sections of the traditional *College Algebra* course and two flipped sections of *College Algebra* using the current textbook, Blitzer's *Algebra and Trigonometry: An Early Functions Approach*, and the MyLabsPlus site. Instructors will administer the SALG survey at the beginning and end of the term to assess and compare student learning in traditional and flipped classrooms.

Stage 2: In spring 2015, all sections of *College Algebra* will be flipped. Any necessary changes to course structure and assignments will be made based on observations and results from teaching the fall 2014 sections and from SALG data. In conjunction with science, computer science, and business faculty, we will develop or choose problems for the course as we move towards creating separate STEM and business sections. In order to assess the impact of adding these problems into the course, two of the sections will incorporate our "big" problems related to the city, and the rest of the sections will use the fall 2014 flipped model. We will continue to administer the SALG to all students at the beginning and end of the semester. These results will inform decisions regarding the course redesign.

Stage 3: In fall 2015, all sections taught will be the complete, newly redesigned *College Algebra: Modeling the City* with the flipped structure and city problems. Some sections will be specifically for STEM students and other sections for business students. There will be different versions of the assessments for the different sections with problems relevant to each discipline. The mathematical course content will be the same for all sections, and will cover all necessary topics to be a transferable course.

As we progress through the stages, we will develop a teaching manual for the course. This manual will contain information on how to run a flipped classroom as well as how to choose and use the city problems. We will create a repository of our Chicago problems. The manual will detail the process of problem creation and the sources of data used, allowing other institutions and instructors to use this model to write questions and problems related to their city.

Technology and Public Policy: Robotics and Embedded Intelligence



Dr. Frank Wattenberg
Professor, Mathematics
frank.wattenberg@usma.edu
Engaging Mathematics
Co-Principal Investigator

Technology is an important partner with mathematics and the sciences as we confront serious public policy questions. This year, for example, with problems between the police and the communities they serve in places like Cleveland, New York City, and Ferguson and with the increasing use of drones in military conflicts, we see that technology can be an important part of the solution or of the problem. It is no accident that science fiction like Asimov's focuses on the same principles that are important in interactions between police and the communities they serve – embodied as Asimov's Three Laws of Robotics:

(1) A robot may not injure a human being or, through inaction, allow a human being to come to harm;

(2) A robot must obey the orders given to it by human beings, except where such orders would conflict with the First Law;

(3) A robot must protect its own existence as long as such protection does not conflict with the First or Second Law.

Asimov was talking about embedded intelligence. Embedded intelligence now surrounds us – for example, machine-learning algorithms determine eligibility for loans.

The author's favorite holiday present is the Arduino kit (pictured, left). This kit and the "Maker" movement empower kids in middle school and above to create and build their own devices with embedded intelligence. These devices can help us understand the problems we face by collecting data and they can help us solve the problems we face by, for example, the "smart" use of energy. But students will be able to exploit the full power of Arduino kits and embedded intelligence only with a solid foundation in mathematics and the sciences and an appreciation for the ethical principles at stake – for example, how much privacy are we willing to trade for security? What decisions are we willing to cede to embedded intelligence? And how do we imbue embedded decision making with an ethical dimension? We are just beginning to use Arduino kits in our classes both to motivate mathematics and to use mathematics to solve pressing and often controversial problems.

CONNECT WITH ENGAGING MATHEMATICS

Leadership

Wm. David Burns
Executive Director, NCSCE
Principal Investigator,
Engaging Mathematics
david.burns@sencer.net

Dr. Cindy Kaus
Co-Principal Investigator
cindy.kaus@metrostate.edu

Dr. Mangala Kothari
Co-Principal Investigator
mkothari@lagcc.cuny.edu

Dr. Frank Wattenberg
Co-Principal Investigator
frank.wattenberg@usma.edu

Christine Marie DeCarlo
Program Coordinator
christine.marie.decarlo@
ncsce.net

Advisory Board

Dr. Chris Arney
Professor, Mathematics
Chair, Network Science
United States
Military Academy

Dr. Prabha Betne
Professor, Mathematics
LaGuardia Community
College

Dr. Victor Donnay
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William R. Kenan, Jr. Chair,
Mathematics
Bryn Mawr College

Dr. David Ferguson
Associate Provost,
Diversity and Inclusion
Distinguished Service
Professor and Chair,
Technology and Society
Stony Brook University

Dr. Susan Forman
Professor, Mathematics and
Computer Science
Bronx Community College

National Center for Science and Civic Engagement (NCSCE)

2000 P Street NW
Suite 210
Washington, DC 20036
(202) 483-4600

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