Groundwater Pollution Module

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This module has been tailored for students in the standard calculus sequence, including pre-calculus, or a mathematics modeling course, and was designed as a portable unit that can be implemented within the standard curriculum as individual or group research projects. The aim is to provide a self-contained introduction to the subject and to engage the students in the process of using mathematical models to investigate real events of ground water pollution.

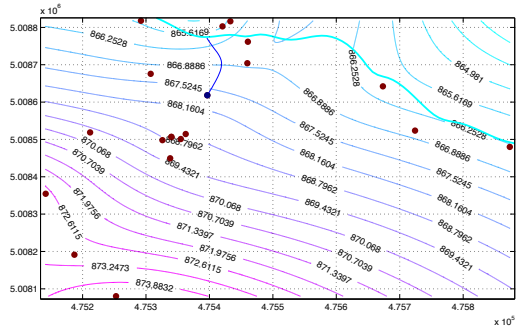

Figure 1: Water table contour map showing a contaminated ground-water flow line from a monitoring well to a nearby creek. This is a MATLAB graphical output from a project based on a municipal landfill located in the city of Andover, Anoka County, Minnesota.

A typical framework to model groundwater is to consider a flow that is essentially horizontal, moving on either of the two directions of an imaginary plane that is "parallel" to the earth's surface, and assume that the height h of water level at any point in the aquifer, known as the *hydraulic head*, is a function of the variables x and y representing the two dimensions of the flow field. The values of h on an aquifer are fundamental to determine the direction and velocity of the flow according to the fundamental Darcy's law (see insert).

This module can be implemented at different levels of instruction. Students in a pre-calculus course can use a discrete version of (1)-(2) to track groundwater pollutants and estimate their traveling time between different locations. In a second semester of calculus students are presented with realistic examples where contaminated ground-water flow lines can be obtained analytically by solving a separable differential equation. More advanced students use numerical methods to handle real events of groundwater pollution (see Fig 1).

Darcy's law, relates the *flux* q, which is the discharge rate per unit cross-sectional area, with the gradient ∇h of the hydraulic head

$$q = -K\nabla h, \tag{1}$$

where K is the *hydraulic conductivity*. The fluid net velocity called *interstitial velocity* or, simply, velocity of the ground water, is given by the following formula

$$v = \frac{-K\nabla h}{\eta}, \tag{2}$$

where η denotes the *porosity* of the medium.